[2] $\mathcal{L}\{t^2 \sin t \sinh t\} = (-1)^2 \dfrac{d^2}{ds^2} \mathcal{L}\{\sin t \sinh t\}$

$$= \frac{d^2}{ds^2} \frac{2s}{s^4+4}$$

$$= \frac{d}{ds} \boxed{\frac{2(s^4+4) - 2s(4s^3)}{(s^4+4)^2}} \quad 1$$

$$= \frac{d}{ds} \boxed{\frac{8-6s^4}{(s^4+4)^2}} \quad \tfrac{1}{2}$$

$$= \boxed{\frac{-24s^3(s^4+4)^{\cancel{2}} - (8-6s^4)2\cancel{(s^4+4)}4s^3}{(s^4+4)^{\cancel{4}\,3}}} \quad 1$$

$$= \frac{-24s^7 - 96s^3 - 64s^3 + 48s^7}{(s^4+4)^3}$$

$$= \boxed{\frac{24s^7 - 160s^3}{(s^4+4)^3}} \quad \tfrac{1}{2}$$

[3]

$$\begin{array}{r|rrrr} 4 & 1 & 0 & 4 & -80 \\ & & 4 & 16 & 80 \\ \hline & 1 & 4 & 20 & \boxed{0} \end{array}$$ $\frac{1}{2}$

$s^2+4s+20=(s+2)^2+16$

$$\frac{2s^2+40s+16}{s^3+4s-80} = \boxed{\frac{\overset{4}{\cancel{A}}}{s-4} + \frac{\overset{-2}{\cancel{B}}(s+2)+\overset{5}{\cancel{C}}(4)}{(s+2)^2+16}}$$ $\frac{1}{2}$

$\frac{1}{2}$

$$\boxed{2s^2+40s+16 = A\left[(s+2)^2+16\right] + B(s+2)(s-4) + C(4)(s-4)}$$

$s=4$: $32+160+16 = A(52) \longrightarrow \boxed{\begin{array}{c} 208 = 52A \\ A=4 \end{array}}$ $\frac{1}{2}$

$s=-2$: $8-80+16 = A(16)+C(4)(-6)$

$\boxed{-56 = 64-24C} \longrightarrow 24C = 120$

$\boxed{C=5}$ $\frac{1}{2}$

$\frac{1}{2}$

s^2: $\boxed{2 = A+B \longrightarrow B = 2-A = -2}$

SANITY CHECK: $s=2$

$\frac{1}{2}$

LHS $\dfrac{8+80+16}{8+8-80} = \dfrac{104}{-64} = -\dfrac{13}{8}$

RHS $\dfrac{4}{-2} + \dfrac{-2(4)+5(4)}{32}$

$= -2+\frac{3}{8} = \frac{-16+3}{8} = \frac{-13}{8}$

TALK TO ME IF YOU USED A DIFFERENT VALUE OF S

$$\mathcal{L}^{-1}\left\{\frac{2s^2+40s+16}{s^3+4s-80}\right\}$$

$$= \boxed{4e^{4t}-2e^{-2t}\cos 4t+5e^{-2t}\sin 4t}$$

(1)

[4] $\int_0^3 e^{-st} e^{-2t}\,dt + \int_3^\infty e^{-st} t^2\,dt$

$t^2 \quad + e^{-st}$

$2t \quad - \frac{1}{s}e^{-st}$

$2 \quad + \frac{1}{s^2}e^{-st}$

$0 \quad - \frac{1}{s^3}e^{-st}$

$$= \frac{1}{2}\ \boxed{\left. -\frac{1}{s+2} e^{-(s+2)t} \right|_0^3} + \boxed{\lim_{N\to\infty} \left. -\left(\frac{1}{s}t^2 + \frac{2}{s^2}t + \frac{2}{s^3}\right)e^{-st} \right|_3^N}\ 1$$

$$= -\frac{1}{s+2}\left(e^{-3(s+2)} - 1\right)$$

$$+ \boxed{\lim_{N\to\infty}\left(-\left(\frac{1}{s}N^2 + \frac{2}{s^2}N + \frac{2}{s^3}\right)e^{-sN} + \left(\frac{9}{s} + \frac{6}{s^2} + \frac{2}{s^3}\right)e^{-3s}\right)}\ \frac{1}{2}$$

$$= \frac{1}{2}\ \boxed{\frac{1 - e^{-3(s+2)}}{s+2}} + \boxed{e^{-3s}\left(\frac{9}{s} + \frac{6}{s^2} + \frac{2}{s^3}\right)}\ \frac{1}{2}$$

$$\lim_{N\to\infty} -\frac{\frac{1}{s}N^2 + \frac{2}{s^2}N + \frac{2}{s^3}}{e^{sN}} \quad \frac{1}{2}$$

$$= \boxed{\lim_{N\to\infty} -\frac{\frac{2}{s}N + \frac{2}{s^2}}{s e^{sN}} = \lim_{N\to\infty} -\frac{\frac{2}{s}}{s^2 e^{sN}}} = 0$$

[5]

$$f(t) = \boxed{\begin{cases} 2+6t, & \text{IF } t<2 \\ 17-3t, & \text{IF } t\geq 2 \end{cases}} \ \frac{1}{2} = 2+6t+u(t-2)(17-3t-(2+6t))$$

$$= \boxed{2+6t+u(t-2)(15-9t)} \ \frac{1}{2}$$

$$\mathcal{L}\{f(t)\} = \frac{2}{s}+\frac{6}{s^2}+e^{-2s}\mathcal{L}\{15-9(t+2)\}$$

$$= \frac{2}{s}+\frac{6}{s^2}+e^{-2s}\mathcal{L}\{\boxed{-3-9t}\} \ \frac{1}{2}$$

$$= \frac{1}{2} \ \boxed{\frac{2}{s}+\frac{6}{s^2}} \ \boxed{-e^{-2s}\left(\frac{3}{s}+\frac{9}{s^2}\right)} \ \frac{1}{2}$$

$$= \frac{2(s+3)}{s^2} - e^{-2s}\frac{3(s+3)}{s^2} = F(s)$$

$$\mathcal{L}\{y''+2y'-3y\} = F(s)$$

$$\frac{1}{2} \ \boxed{\begin{array}{l} s^2\mathcal{L}\{y\} - s\,\cancel{y(0)}^{-1} - \cancel{y'(0)}^{3} \\ +2(s\mathcal{L}\{y\} \qquad - \cancel{y(0)}^{-1}) \\ -3\mathcal{L}\{y\} \end{array}} = F(s)$$

$$\frac{1}{2} \ \boxed{(s^2+2s-3)\mathcal{L}\{y\}+s-1} = F(s)$$

$$\mathcal{L}\{y\} = \frac{-(s-1)}{s^2+2s-3} + \frac{F(s)}{s^2+2s-3}$$

$$\mathcal{L}\{y\} = \frac{-(s-1)}{(s+3)(s-1)} + \frac{2(s+3)}{s^2(s+3)(s-1)} - e^{-2s}\frac{3(s+3)}{s^2(s+3)(s-1)}$$

$$= \boxed{-\frac{1}{s+3} + \frac{2}{s^2(s-1)} - e^{-2s}\frac{3}{s^2(s-1)}} \; 1$$

1

$$y = 1 \; \boxed{-e^{-3t} + 2(-1-t+e^{t})} \; \boxed{-3u(t-2)(-1-(t-2)+e^{t-2})}$$

$$= -e^{-3t} - 2 - 2t + 2e^{t} + u(t-2)(-3+3t-3e^{t-2})$$

$$\frac{1}{s^2(s-1)} = \boxed{\frac{\cancel{A}^{\,-1}}{s} + \frac{\cancel{B}^{\,-1}}{s^2} + \frac{\cancel{C}^{\,1}}{s-1}} \; \frac{1}{2}$$

$$\boxed{1 = As(s-1) + B(s-1) + Cs^2} \; \frac{1}{2}$$

$$\boxed{\begin{aligned} s=0: &\quad 1 = B(-1) \longrightarrow B = -1 \\ s=1: &\quad 1 = C \\ s^2: &\quad 0 = A + C \longrightarrow A = -C = -1 \end{aligned}} \; 1$$

SANITY CHECK: $s = 2$

$$\boxed{\begin{aligned} \text{LHS} &\quad \frac{1}{4(1)} = \frac{1}{4} \\ \text{RHS} &\quad \frac{-1}{2} - \frac{1}{4} + \frac{1}{1} = \frac{1}{4} \end{aligned}} \; \frac{1}{2}$$